Three-dimensional analysis of steady groundwater flow near ellipsoidal lakes

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Introduction

The management of human activities in areas characterized by environmentally sensitive lakes and wetlands is problematic, especially where groundwaters are contaminated. Groundwater capture and release zones for surface water bodies (Townley & Davidson 1988) are important for water resource managers, and have recently been studied for two-dimensional (Nield et al. 1994, Smith 1999) and three-dimensional (Townley & Trefry 2000) lake-aquifer systems.

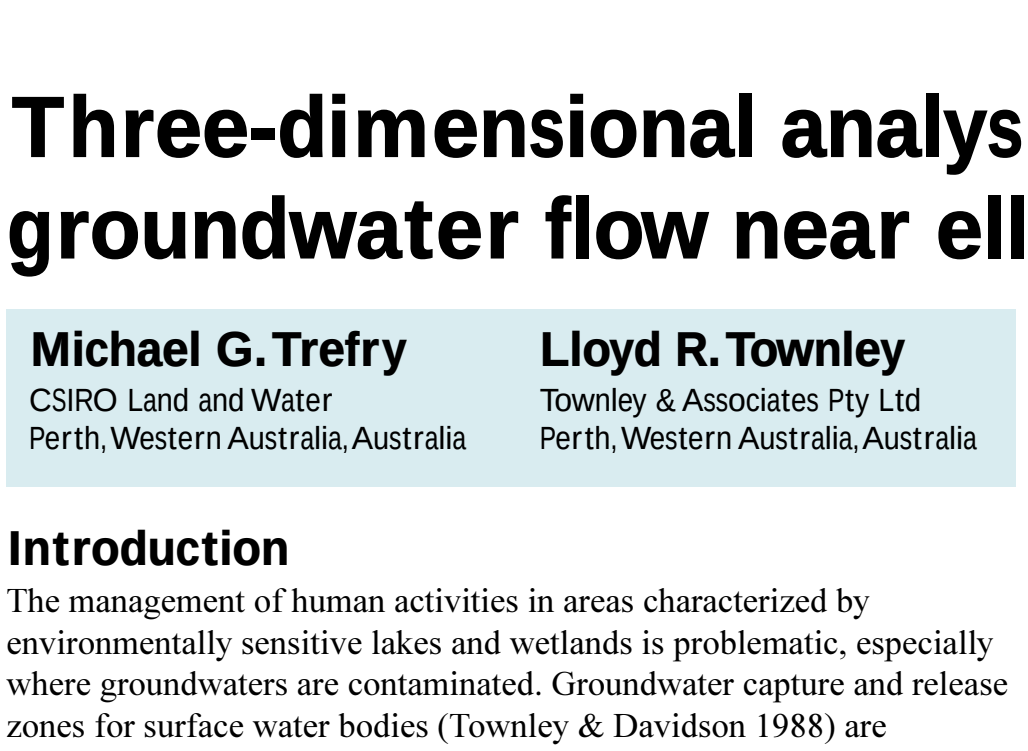

Photo Bill van Aken, CSIRO Land and Water

Figure 1. Aerial photograph of lake systems in a semi-urban setting. Photo of Lake Yangebup, 20 km south of Perth, Western Australia, looking north-west to the Indian Ocean.

Here these shallow lake results are extended to cases where the lakes have half-ellipsoidal geometry, i.e. elliptical intersection with the ground surface and variable depth profiles.

Capture Zones for Ellipsoidal Lakes

Figure 2 shows a schematic of a lake-aquifer system and symmetric capture/release zones where regional groundwater flow is aligned with the x axis. Capture zone width (w) and depth (d) parameters are of interest. The lake ellipsoid is centered at coordinates $(x,y,z) = (0,0,0)$ and has semi-axes (a,b,c). The mirror plane at $z = 0$ corresponds to a horizontal water table, which is an approximation to a gently sloping water table with zero vertical recharge. Regional throughflow is assumed to be more significant than local recharge. The aquifer half-space is assumed to be fully saturated, uniform, homogeneous and isotropic. A discussion of these assumptions is found in Townley & Trefry (2000). Trefry & Townley (1998) derived an analytical potential for steady flow near embedded ellipsoids from earlier results (Lamb 1945, Carslaw & Jaeger 1959). This system is analogous to that of Figure 2. Using this potential, aggregations of flow paths calculated via particle tracking techniques give the 3D capture/release zone diagrams of Figure 3.

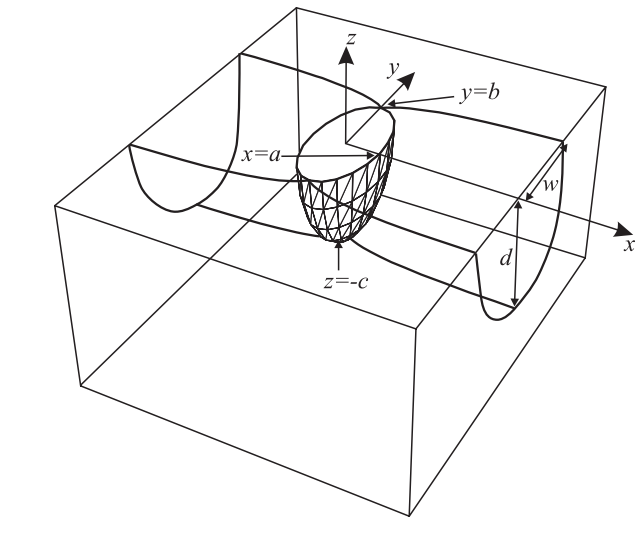

Figure 2. Schematic half-ellipsoidal lake-aquifer system, showing symmetric capture and release zones. The lakebed is drawn as a hatched surface.

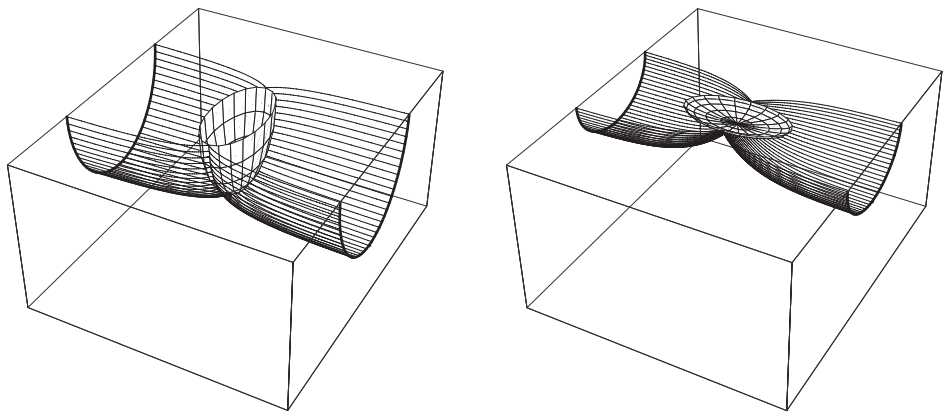

Figure 3. Symmetric capture/release zone for two half-ellipsoidal lakes with semiaxes $(a,b,c) = (1,2,3)$ (left) and $(2,1,1/3)$ (right). Lengths are in arbitrary units.

Figure 4 shows a family of transition curves, representing capture/release zone widths and depths, determined using the approach of Trefry & Townley (1998). Problem symmetry ensures that the same curves may be used for both w and d merely by interchanging b and c values. The curves map the evolution of the capture zone parameters as the lake ellipsoid varies over wide ranges of shape and orientation with respect to regional flow. Full algorithm details are available on request. The capture/release zone transition curves provide water resource managers with a simple means of mapping results from groundwater flow regime theory for restricted lake geometries to more realistic three-dimensional lake-aquifer systems.

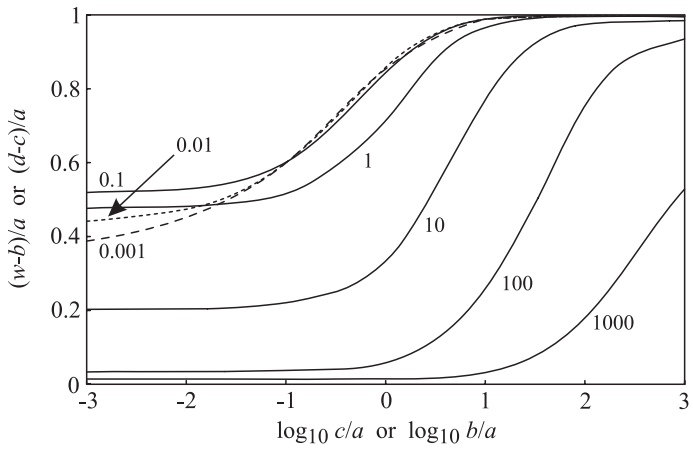

Figure 4. Transition curves for reduced widths $(w-b)/a$ versus $\log_{10} c/a$ for indicated values of b/a, or for reduced depths $(d-c)/a$ versus $\log_{10} b/a$ for indicated values of c/a.

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